

Aufgaben zu Exponential- und Logarithmusgleichungen

1.0 Lösen Sie folgende Exponentialgleichungen durch Probieren.

1.1 $3^x = 729$

1.2 $2^x = 1024$

1.3 $2^x = \frac{1}{512}$

1.4 $6^x = 1$

1.5 $1,5^x = 3,375$

1.6 $\left(\frac{5}{2}\right)^x = \frac{2}{5}$

1.7 $\left(\frac{3}{4}\right)^{2x} = \frac{27}{64}$

1.8 $\left(\frac{1}{2}\right)^{-2x} = 512$

2.0 Lösen Sie folgende Exponentialgleichungen durch Exponentenvergleich.

2.1 $2^5 \cdot 2^{x-1} = 16$

2.2 $2^{3x-1} = 4^{x+1}$

2.3 $4^{3x-1} \cdot 2^{4x} = 8^{-x+4}$

2.4 $10^{-2(x+1)} = 0,01^{5x+3}$

2.5 $\frac{1}{64} = 8^{4x+0,5}$

2.6 $\left(\frac{1}{4}\right)^{x+7} = 2^{2x}$

3.0 Lösen Sie folgende Exponentialgleichungen durch Substitution.

3.1 $2 \cdot 5^{2x} - 52 \cdot 5^x = -50$

3.2 $2^{2x} - 12 \cdot 2^x + 32 = 0$

3.3 $10^{2x} - 100,1 \cdot 10^x = -10$

4.0 Lösen Sie folgende Exponentialgleichungen auf 4 Dezimalstellen nach dem Komma.

4.1 $6^x = 44$

4.2 $\left(\frac{1}{2}\right)^x = 0,2$

4.3 $7^{2x} = 100$

4.4 $2^{x-1} = 35$

4.5 $0,05^{-x} = 9$

4.6 $2^{-x+1} = 398$

4.7 $5^{3x-3} = 200$

4.8 $0,04^{2x-1} = 10$

5.0 Zerlegen Sie folgende Terme.

5.1 $\log_b(x^3 y^2 z^4)$

5.2 $\log_b 4x^2(x-y)^3$

5.3 $\log_b \frac{5m^3 n^4}{3p^2}$

5.4 $\log_b \frac{2\sqrt[4]{q}}{3}$

5.5 $\log_b(3x^3 \cdot \frac{5y}{6z})$

5.6 $\log_b \frac{1}{r^2 s^3 t^4}$

6.0 Fassen Sie folgende Terme zusammen.

6.1 $\frac{1}{2} \cdot \log_b x^3 - \log_b x - 3 \cdot \log_b x$

6.2 $m \cdot \log_b u + \frac{n}{3} \cdot \log_b v - \frac{m}{n} \cdot \log_b w$

6.3 $-5 \cdot \log_b b + \log_b 1 + \frac{1}{2} \cdot \log_b b^3$

6.4 $3 \cdot \log_b(x+y) + 2 \cdot \log_b(x-y) - 4 \cdot \log_b(x^2 - y^2)$

$$6.5 \frac{1}{3} \cdot \log_b(x+y) - \frac{1}{3} \cdot \log_b(x^2 - y^2) - \frac{1}{2} \cdot \log_b(x-y) + \log_b(x-y)$$

$$6.6 \frac{1}{6} \cdot (5 \cdot \log_b u - \log_b v^2) + \frac{1}{4} \cdot (\log_b w - \log_b z)$$

7.0 Bestimmen Sie die Definitionsmenge und lösen Sie folgende Logarithmusgleichungen.

$$7.1 \log_4 x = 0,5 \quad 7.2 \log_{0,5} x = 4 \quad 7.3 \log_{\sqrt{2}} x = 6 \quad 7.4 \log_{\frac{3}{4}} x = 2$$

$$7.5 \log_{\frac{2}{3}} x = -4 \quad 7.6 \log_5(x-3) = 10 \quad 7.7 \log_7(4-x) = 80 \quad 7.8 \ln(x+3) = 7,56$$

$$7.9 \log_4(2x+6) - \log_4(x-2) = 4 \quad 7.10 \log_3(x-3) - \log_3(x-5) = \log_3(2x-8)$$

$$7.11 \frac{1}{2} \cdot \ln 2x - \ln \frac{x}{2} = 0 \quad 7.12 \lg(x-8) + \lg(x+2) = \lg(2x+4)$$

$$7.13 \log_a x^3 + \log_a x - \log_a x^2 = 0 \quad 7.14 \log_4 x^3 = 3 + \log_4 2$$

$$7.15 \log_7(x-2) + \log_7(x+2) - \log_7(3x-10) = \log_7(x-2)$$

$$7.16 \frac{1}{4} \cdot \log_3 x - \log_3 x^2 = \log_3 \sqrt[3]{2} - \frac{1}{3} \cdot \log_3 x - \log_3 4^3$$

8.0 Lösen Sie folgende Aufgaben mit Hilfe der Musteraufgabe.

Musteraufgabe: $2^{x+1} + 2^{x+2} = 3 \Rightarrow 2^x \cdot 2 + 2^x \cdot 2^2 = 3 \Rightarrow 2^x(2+2^2) = 3 \Rightarrow 2^x \cdot 6 = 3$
 $\Rightarrow 2^x = \frac{1}{2} \Rightarrow x = -1$

$$8.1 2^{x+1} + 2^x = 24 \quad 8.2 3^{2x+2} - 3^{2x+1} + 3^{2x} = 14 \quad 8.3 4^x - 2^{2x+1} + 4^{x+2} = 120$$

9.0 Lösen Sie folgende Aufgaben mit Hilfe der Musteraufgabe.

(Exponentialgleichung mit verschiedenen Basen)

Musteraufgabe: $3^{2x+1} - 2^{x+1} = 2^{x+2} - 9^{x+1} \Rightarrow 3^{2x+1} - 2^{x+1} = 2^{x+2} - 3^{2(x+1)}$
 $\Rightarrow 3^{2x+1} + 3^{2x+2} = 2^{x+2} + 2^{x+1} \Rightarrow 3^{2x}(3+3^2) = 2^x(2^2+2)$
 $\Rightarrow 12 \cdot 3^{2x} = 6 \cdot 2^x \Rightarrow 2 \cdot 3^{2x} = 2^x \Rightarrow \lg(2 \cdot 3^{2x}) = \lg 2^x$
 $\Rightarrow \lg 2 + 2x \cdot \lg 3 = x \cdot \lg 2 \Rightarrow 2x \cdot \lg 3 - x \cdot \lg 2 = -\lg 2$
 $\Rightarrow x(2 \cdot \lg 3 - \lg 2) = -\lg 2 \Rightarrow x = \frac{-\lg 2}{(2 \lg 3 - \lg 2)} \approx -0,461$

$$9.1 3^{x+1} + 3^x = 2^{x+2} \quad 9.2 5^x + 6^x = 6^{x+1} \quad 9.3 4 \cdot 5^x - 2^{2x} = 4^{x+1}$$

$$9.4 24 \cdot 8^{x-1} - 27^{x+1} = 2^{3x} - 24 \cdot 3^{3x} \quad 9.5 25^{x+1} - 18 \cdot 9^{x-1} = 20 \cdot 5^{2x} + 3^{2x}$$

Lösungen

1.1 $x = 6$ 1.2 $x = 10$ 1.3 $x = -9$ 1.4 $x = 0$

1.5 $x = 3$ 1.6 $x = -1$ 1.7 $x = \frac{3}{2}$ 1.8 $x = \frac{9}{2}$

2.1 $2^{5+x-1} = 2^4 \Rightarrow 5+x-1=4 \Rightarrow x=0$

2.2 $2^{3x-1} = 2^{2(x+1)} \Rightarrow 3x-1=2x+2 \Rightarrow x=3$

2.3 $2^{2(3x-1)+4x} = 2^{3(-x+4)} \Rightarrow 6x-2+4x=-3x+12 \Rightarrow x = \frac{14}{13}$

2.4 $10^{-2(x+1)} = 10^{-2(5x+3)} \Rightarrow -2(x+1)=-2(5x+3) \Rightarrow x = -\frac{1}{2}$

2.5 $8^{-2} = 8^{4x+0,5} \Rightarrow -2=4x+0,5 \Rightarrow x = -\frac{5}{8}$

2.6 $2^{-2(x+7)} = 2^{2x} \Rightarrow -2x-14=2x \Rightarrow x = -\frac{7}{2}$

3.1 Substitution: $z = 5^x$

$$\Rightarrow 2z^2 - 52z = -50 \Rightarrow 2z^2 - 52z + 50 = 0 \Rightarrow z = 25 \quad \text{und} \quad z = 1$$

Resubstitution: $5^x = 25 \Rightarrow x = 2$

$$5^x = 1 \Rightarrow x = 0$$

3.2 Substitution: $z = 2^x$

$$\Rightarrow z^2 - 12z + 32 = 0 \Rightarrow z = 8 \quad \text{und} \quad z = 4$$

Resubstitution: $2^x = 8 \Rightarrow x = 3$

$$2^x = 4 \Rightarrow x = 2$$

3.3 Substitution: $z = 10^x$

$$\Rightarrow z^2 - 100,1z = -10 \Rightarrow z^2 - 100,1z + 10 = 0 \Rightarrow z = 100 \quad \text{und} \quad z = \frac{1}{10}$$

Resubstitution: $10^x = 100 \Rightarrow x = 2$

$$10^x = \frac{1}{10} \Rightarrow x = -1$$

4.1 $6^x = 44 \Rightarrow x \cdot \lg 6 = \lg 44 \Rightarrow x = \frac{\lg 44}{\lg 6} \approx 2,1120$

4.2 $x \approx 2,3219$ 4.3 $x \approx 1,1833$ 4.4 $(x-1) \cdot \lg 2 = \lg 35 \Rightarrow x = \frac{\lg 35}{\lg 2} + 1 \approx 6,1293$

$$4.5 \quad x \approx 0,7335 \quad 4.6 \quad x = 1 - \frac{\lg 398}{\lg 2} \approx -7,6366 \quad 4.7 \quad x = \frac{1}{3} \cdot \left(3 + \frac{\lg 200}{\lg 5} \right) \approx 2,0973$$

$$4.8 \quad x = \frac{1}{2} \cdot \left(\frac{\lg 10}{\lg 0,04} + 1 \right) \approx 0,1423$$

$$5.1 \quad \log_b x^3 + \log_b y^2 + \log_b z^4 = 3 \cdot \log_b x + 2 \cdot \log_b y + 4 \cdot \log_b z$$

$$5.2 \quad \log_b 4 + \log_b x^2 + \log_b (x-y)^3 = 2 \cdot \log_b 2 + 2 \cdot \log_b x + 3 \cdot \log_b (x-y)$$

$$5.3 \quad \log_b 5m^3n^4 - \log_b 3p^2 = \log_b 5 + 3 \cdot \log_b m + 4 \cdot \log_b n - \log_b 3 - 2 \cdot \log_b p$$

$$5.4 \quad \log_b 2\sqrt[4]{q} - \log_b 3 = \log_b 2 + \frac{1}{4} \cdot \log_b q - \log_b 3$$

$$5.5 \quad \log_b 3x^3 + \log_b 5y - \log_b 6z = \log_b 3 + 3 \cdot \log_b x + \log_b 5 + \log_b y - \log_b 6 - \log_b z$$

$$5.6 \quad \log_b 1 - \log_b r^2 s^3 t^4 = 0 - 2 \cdot \log_b r - 3 \cdot \log_b s - 4 \cdot \log_b t$$

$$6.1 \quad \frac{3}{2} \cdot \log_b x - 4 \cdot \log_b x = -\frac{5}{2} \cdot \log_b x = \log_b x^{-\frac{5}{2}} = \log_b \frac{1}{\sqrt{x^5}}$$

$$6.2 \quad \log_b \frac{u^m \cdot v^{\frac{n}{3}}}{w^{\frac{m}{n}}} = \log_b \frac{u^m \cdot \sqrt[3]{v^n}}{\sqrt[n]{w^m}} \quad 6.3 \quad -5 + 0 + \frac{3}{2} \cdot \log_b b = -\frac{7}{2}$$

$$6.4 \quad \log_b (x+y)^3 + \log_b (x-y)^2 - \log_b (x^2 - y^2)^4 = \log_b \frac{(x+y)^3 (x-y)^2}{(x^2 - y^2)^4} =$$

$$\log_b \frac{(x+y)^3 (x-y)^2}{(x-y)^4 (x+y)^4} = \log_b \frac{1}{(x+y)(x-y)^2}$$

$$6.5 \quad \frac{1}{3} \cdot \log_b \frac{x+y}{(x+y)(x-y)} + \frac{1}{2} \cdot \log_b (x-y) = -\frac{1}{3} \cdot \log_b (x-y) + \frac{1}{2} \cdot \log_b (x-y) =$$

$$\frac{1}{6} \cdot \log_b (x-y) = \log_b \sqrt[6]{x-y}$$

$$6.6 \quad \frac{1}{6} \cdot \log_b \frac{u^5}{v^2} + \frac{1}{4} \cdot \log_b \frac{w}{z} = \log_b \sqrt[6]{\frac{u^5}{v^2}} \cdot \sqrt[4]{\frac{w}{z}}$$

$$7.1 \quad D = \mathbb{R}^+ \quad x = \sqrt{2^6} = 2^3 = 8 \quad 7.2 \quad D = \mathbb{R}^+ \quad x = 0,5^4 = \frac{1}{16}$$

$$7.3 \quad D = \mathbb{R}^+ \quad x = \sqrt{2^6} = 2^3 = 8 \quad 7.4 \quad D = \mathbb{R}^+ \quad x = \left(\frac{3}{4} \right)^2 = \frac{9}{16}$$

$$7.5 \quad D = \mathbb{R}^+ \quad x = \left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16} \quad 7.6 \quad D =]3; \infty[\quad x - 3 = 5^{10} \Rightarrow x = 5^{10} + 3$$

$$7.7 \quad D =]-\infty; 4[\quad 4 - x = 7^{80} \Rightarrow x = 4 - 7^{80}$$

$$7.8 \quad D =]-3; \infty[\quad x + 3 = e^{7,56} \Rightarrow x = e^{7,56} - 3 \approx 1916,8455$$

$$7.9 \quad D =]2; \infty[\quad \log_4 \frac{2x+6}{x-2} = 4 \Rightarrow \frac{2x+6}{x-2} = 4^4 \Rightarrow x = \frac{259}{127} \approx 2,0394$$

$$7.10 \quad D =]5; \infty[$$

$$\log_3 \frac{x-3}{x-5} = \log_3 (2x-8) \Rightarrow \frac{x-3}{x-5} = 2x-8 \Rightarrow 2x^2 - 19x + 43 = 0$$

$$\Rightarrow x = \frac{19 \pm \sqrt{17}}{4} \Rightarrow x \approx 3,7192 \quad \text{und} \quad x \approx 5,7808 \quad \Rightarrow L = \{5,7808\}$$

$$7.11 \quad D = \mathbb{R}^+ \quad \ln \frac{\sqrt{2x}}{\frac{x}{2}} = 0 \Rightarrow \frac{\sqrt{2x}}{\frac{x}{2}} = 1 \Rightarrow \sqrt{2x} = \frac{x}{2} \Rightarrow 2x = \frac{x^2}{4} \Rightarrow x = 8 \quad \text{und}$$

$$x = 0 \quad \Rightarrow L = \{8\}$$

$$7.12 \quad D =]8; \infty[\quad \lg(x-8)(x+2) = \lg(2x+4) \Rightarrow (x-8)(x+2) = 2x+4$$

$$\Rightarrow x = 10 \quad \text{und} \quad x = -2 \quad \Rightarrow L = \{10\}$$

$$7.13 \quad D = \mathbb{R}^+ \quad 3 \cdot \log_a x + \log_a x - 2 \cdot \log_a x = 0 \Rightarrow 2 \cdot \log_a x = 0 \Rightarrow x = 1$$

$$7.14 \quad D = \mathbb{R}^+ \quad \log_4 x^3 = 3 \cdot \log_4 4 + \log_4 2 \Rightarrow \log_4 x^3 = \log_4 4^3 \cdot 2$$

$$\Rightarrow x^3 = 128 \Rightarrow x = \sqrt[3]{128}$$

$$7.15 \quad D = \left] \frac{10}{3}; \infty \right[\quad \log_7 \frac{x+2}{3x-10} = 0 \Rightarrow \frac{x+2}{3x-10} = 1 \Rightarrow x = 6$$

$$7.16 \quad D = \mathbb{R}^+ \quad \log_3 \frac{x^4}{x^2} = \log_3 \frac{\sqrt[3]{2}}{\sqrt[3]{x \cdot 4^3}} \Rightarrow x^{\frac{7}{4}} = \frac{\sqrt[3]{2}}{x^{\frac{1}{3}} \cdot 4^3} \Rightarrow x^{\frac{17}{12}} = \frac{\sqrt[3]{2}}{64}$$

$$x = \left(\frac{64}{\sqrt[3]{2}} \right)^{\frac{12}{17}} = 16$$

$$8.1 \quad 2^x(2+1) = 24 \Rightarrow 2^x = 8 \Rightarrow x = 3$$

$$8.2 \quad 3^{2x}(3^2 - 3 + 1) = 14 \Rightarrow 3^{2x} = 2 \Rightarrow 2x = \frac{\lg 2}{\lg 3} \Rightarrow x = \frac{1}{2} \cdot \frac{\lg 2}{\lg 3} \approx 0,3155$$

$$8.3 \quad 2^{2x} - 2^{2x+1} + 2^{2x+4} = 120 \Rightarrow 2^{2x}(1 - 2 + 2^4) = 120 \Rightarrow 2^{2x} = 8 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$$

$$9.1 \quad 3^x(3+1) = 4 \cdot 2^x \Rightarrow 3^x = 2^x \Rightarrow x \cdot \lg 3 - x \cdot \lg 2 = 0 \Rightarrow x \cdot (\lg 3 - \lg 2) = 0 \\ \Rightarrow x = 0$$

$$9.2 \quad 5^x + 6^x(1-6) = 0 \Rightarrow 5^x = 5 \cdot 6^x \Rightarrow x \cdot \lg 5 = \lg 5 + x \cdot \lg 6 \\ \Rightarrow x \cdot (\lg 5 - \lg 6) = \lg 5 \Rightarrow x = \frac{\lg 5}{\lg 5 - \lg 6} \approx -8,8275$$

$$9.3 \quad 4 \cdot 5^x = 2^{2x+2} + 2^{2x} \Rightarrow 4 \cdot 5^x = 2^{2x}(2^2 + 1) \Rightarrow 5^x = \frac{5}{4} \cdot 2^{2x} \\ \Rightarrow x \cdot \lg 5 = \lg \frac{5}{4} + 2x \cdot \lg 2 \Rightarrow x \cdot (\lg 5 - 2 \lg 2) = \lg \frac{5}{4} \\ \Rightarrow x = \frac{\lg \frac{5}{4}}{\lg 5 - 2 \lg 2} = 1$$

$$9.4 \quad 24 \cdot 2^{3x-3} - 3^{3x+3} = 2^{3x} - 24 \cdot 3^{3x} \Rightarrow 24 \cdot 2^{3x-3} - 2^{3x} = 3^{3x+3} - 24 \cdot 3^{3x} \\ 2^{3x}(24 \cdot 2^{-3} - 1) = 3^{3x}(27 - 24) \Rightarrow 2 \cdot 2^{3x} = 3 \cdot 3^{3x} \\ \Rightarrow \lg 2 + 3x \cdot \lg 2 = \lg 3 + 3x \cdot \lg 3 \\ \Rightarrow 3x(\lg 2 - \lg 3) = \lg 3 - \lg 2 \Rightarrow x = \frac{1}{3} \cdot \frac{\lg 3 - \lg 2}{\lg 2 - \lg 3} = -\frac{1}{3}$$

$$9.5 \quad 5^{2x+2} - 18 \cdot 3^{2x-2} = 20 \cdot 5^{2x} + 3^{2x} \Rightarrow 5^{2x+2} - 20 \cdot 5^{2x} = 3^{2x} + 18 \cdot 3^{2x-2} \\ \Rightarrow 5^{2x}(25 - 20) = 3^{2x}(1 + 18 \cdot 3^{-2}) \Rightarrow 5 \cdot 5^{2x} = 3 \cdot 3^{2x} \Rightarrow 5^{2x} = \frac{3}{5} \cdot 3^{2x} \\ \Rightarrow 2x \cdot \lg 5 = \lg \frac{3}{5} + 2x \cdot \lg 3 \Rightarrow 2x(\lg 5 - \lg 3) = \lg \frac{3}{5} \Rightarrow x = \frac{1}{2} \cdot \frac{\lg \frac{3}{5}}{\lg 5 - \lg 3} = -\frac{1}{2}$$